Version 1.0

General Certificate of Education (A-level)
June 2011

# Mathematics MM04

# (Specification 6360)

# Mechanics 4

# Final

# Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk



Set and published by the Assessment and Qualifications Alliance.

The Assessment and Qualifications Alliance (AQA) is a company limited by guarantee registered in England and Wales (company number 3644723) and a registered charity (registered charity number 1073334).
Registered address: AQA, Devas Street, Manchester M15 6EX.

**Key to mark scheme abbreviations**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| $-x$ EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

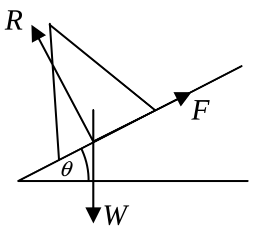
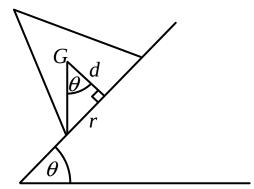
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

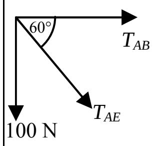
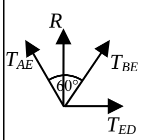
MM04

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 1(a) | $\begin{pmatrix}-1\\1\\0\end{pmatrix}+\begin{pmatrix}4\\0\\-2\end{pmatrix}+\begin{pmatrix}-3\\-1\\2\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ | B1 | 1 | Clear use of $\sum \mathbf{F}_i = \mathbf{0}$ |
| (b) | $\mathbf{r}_1\times\mathbf{F}_1=\begin{vmatrix}\mathbf{i}&0&-1\\\mathbf{j}&2&1\\\mathbf{k}&1&0\end{vmatrix}=\begin{pmatrix}-1\\-1\\2\end{pmatrix}$ | M1 | | Any attempt at $\mathbf{r}\times\mathbf{F}$ or $\mathbf{F}\times\mathbf{r}$ |
| | $\mathbf{r}_2\times\mathbf{F}_2=\begin{vmatrix}\mathbf{i}&3&4\\\mathbf{j}&-1&0\\\mathbf{k}&0&-2\end{vmatrix}=\begin{pmatrix}2\\6\\4\end{pmatrix}$ | | | |
| | $\mathbf{r}_3\times\mathbf{F}_3=\begin{vmatrix}\mathbf{i}&4&-3\\\mathbf{j}&0&-1\\\mathbf{k}&-5&2\end{vmatrix}=\begin{pmatrix}-5\\7\\-4\end{pmatrix}$ | A3,2,1 | | $-1$ each 'type' of error |
| | $\begin{pmatrix}-1\\-1\\2\end{pmatrix}+\begin{pmatrix}2\\6\\4\end{pmatrix}+\begin{pmatrix}-5\\7\\-4\end{pmatrix}=\begin{pmatrix}-4\\12\\2\end{pmatrix}$ | m1<br>A1F | 6 | Summing 3 vector product expressions<br>Follow through their three answers for $\mathbf{r}\times\mathbf{F}$<br>$\mathbf{F}\times\mathbf{r}$ loses final A1 |
| (c) | Resultant force $=\begin{pmatrix}0\\0\\0\end{pmatrix}$ and moment about<br>$O$ is non zero $\Rightarrow$ couple | E2,1 | 2 | E1 $\sum \mathbf{F} = 0$<br>E1 Moment $\neq 0$ |
| | **Total** | | **9** | |

**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **2(a)** | Volume of cone $=\frac{1}{3}\pi r^2 h=\frac{8h^3\pi}{3}$ | B1 | | Correct volume |
| | **Alternative:** $\left[\pi\int_0^{8h}\left(\frac{1}{8}x\right)^2 dx=\frac{8h^3}{3}\pi\right]$ | (B1) | | |
| | $\pi\int xy^2 dx=\pi\int_0^{8h} x\left(\frac{x}{8}\right)^2 dx=\pi\int_0^{8h}\frac{x^3}{64}dx$ | M1 | | Use of $\int xy^2 dx$ |
| | $=\left[\frac{x^4}{256}\right]_0^{8h}$ | A1 | | Correct integration of $kx^3$ |
| | $=16h^4$ | | | |
| | $\bar{x}=\frac{\pi\int xy^2 dx}{\pi\int y^2 dx}=\frac{16h^4\pi}{8h^3\frac{\pi}{3}}$ | M1 | | Attempt at formula for $\bar{x}$ |
| | $=6h$ | A1 | 5 | AG |
| **(b)(i)** |  | B1 | | 3 forces |
| | | B1 | 2 | Through a common point at base of cone |
| **(ii)** |  *G* above point of rotation | | | |
| | $\tan\theta=\frac{r}{d}$ | M1 | | $\tan\theta$ seen anywhere |
| | $=\frac{h}{8h-6h}=\frac{1}{2}$ | A1 | | Ratio correct |
| | $\theta=\tan^{-1}\frac{1}{2}=26.6°$ | A1F | 3 | Use of $\tan^{-1}$ to get $\theta$; ft incorrect ratio |
| **(c)** | Resolve perp to plane $N=W\cos\alpha$; parallel to plane $F=W\sin\alpha$ | B1 | | Both correct |
| | Limiting friction $F=\mu N\Rightarrow\tan\alpha=\frac{6}{13}$ | M1 | | Eliminate $W$, obtain $\tan\alpha$ |
| | $\alpha=24.8°<26.6°$ or $\frac{6}{13}<\frac{1}{2}$ | A1 | | Comparison |
| | $\Rightarrow$ slides first | A1F | 4 | Conclusion; ft (b)(ii) |
| | **Total** | | **14** | |

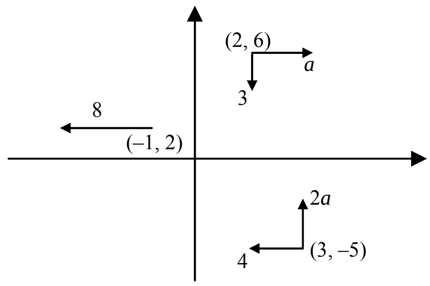
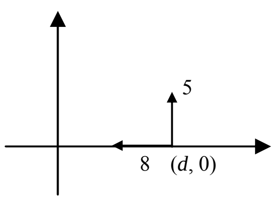
**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)** | System is symmetrical about vertical line through $B$, hence reactions are equal | E1 | 1 | |
| **(b)** | Resolve vertically $2R = 250$<br>$R = 125$ N | M1<br>A1 | <br>2 | Attempt to resolve |
| **(c)** | <br>At $A$ resolve vertically<br>$T_{AE}\sin 60° + 100 = 0$<br>$\lvert T_{AE} \rvert = \dfrac{100}{\sin 60°} = 115.4(\ldots) = 115\text{N}$ | <br>M1<br>A1 | | <br>Attempt at equation involving $AE$<br>AG $\left[\text{or } \dfrac{200\sqrt{3}}{3} \text{ seen}\right]$ |
| | At $A$ resolve horizontally<br>$T_{AB} + T_{AE}\cos 60° = 0$<br>$\lvert T_{AB} \rvert = \dfrac{100}{\sin 60°} \times \cos 60° = 57.73(\ldots) = 57.7\text{N}$ | <br>M1<br>A1 | <br><br>4 | <br>Attempt at equation involving $AB$<br>AG $\left[\text{or } \dfrac{100\sqrt{3}}{3} \text{ seen}\right]$ |
| **(d)** | By symmetry, $T_{BE} = T_{BD} = T$<br>Resolve vertically at $B$: $2T\cos 30° + 50 = 0$<br>$\lvert T_{BE} \rvert = \dfrac{25}{\cos 30°} = 28.86 = 28.9\,\text{N}$ | M1<br>A1<br>A1 | <br><br>3 | Use of symmetry<br><br>Accept $\dfrac{50\sqrt{3}}{3}$ |
| | **Alternative:** At $E$ resolve vertically<br><br>$R + T_{AE}\cos 30° + T_{BE}\cos 30° = 0$<br>$125 - \dfrac{100}{\sin 60°}\cos 30° + T_{BE}\cos 30° = 0$<br>$\lvert T_{BE} \rvert = 28.86 = 28.9\text{N}$ | <br>(M1)<br>(A1)<br>(A1) | <br><br><br>(3) | <br>3 term equation involving $R, T_{AE}, T_{BE}$<br>Correct equation, +/– signs and values correct<br>Accept $\dfrac{50\sqrt{3}}{3}$ |
| **(e)** | At $E$ resolve horizontally<br>$T_{ED} + T_{BE}\cos 60° = T_{AE}\cos 60°$<br>$\lvert T_{ED} \rvert = \dfrac{25}{\cos 30°}\cos 60° - \dfrac{100}{\sin 60°}\cos 60° =$<br>$43.3(\ldots) = 43.3\text{N}$<br>$= 43.3\text{N}$ | <br>M1<br>A1<br><br>A1F | <br><br><br><br>3 | <br>3 term equation including $T_{ED}, T_{BE}, T_{AE}$<br>Correct equation, +/– signs and values correct<br><br>Follow through $T_{AE}$; accept $\dfrac{75\sqrt{3}}{3}$ |
| | **Total** | | **13** | |

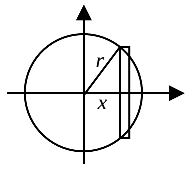
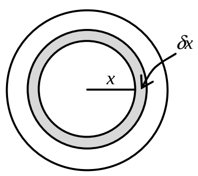
**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a)** | $\text{MI}_{\text{DISC}} = \frac{1}{2}mr^2 = \frac{1}{2}(1.5)(0.2)^2$ | M1 | | Use of $\frac{1}{2}mr^2$ |
| | $= 0.03\,\text{kg m}^2$ | A1 | 2 | AG |
| **(b)(i)** | $V = r\omega$ hence $2 = 0.2\omega$ | M1 | | |
| | $\omega = 10\ \text{rad s}^{-1}$ | A1 | 2 | Attempt to use $r\omega$ |
| **(ii)** | KE gained by pulley $= \frac{1}{2}I\omega^2$ | | | |
| | $= \frac{1}{2}(0.03)(10)^2$ | M1 | | Attempt at KE for pulley |
| | $= 1.5\,\text{J}$ | A1 | | Correct KE for pulley |
| | Conservation of energy:<br>Gain $\text{KE}_{\text{PULLEY}} + \text{KE}_{\text{BUCKET}}$<br>= loss in PE for bucket | | | |
| | $1.5 + \frac{1}{2}(1)(2)^2 = 1gd$ | M1 | | 3 term equation |
| | $d = \frac{5}{14}$ | A1 | 4 | AG |
| **(iii)** | $g - T = 0.2\ddot{\theta}$ | M1A1 | | Equation of motion for bucket $mg - T = mr\ddot{\theta}$ |
| | $0.2T = 0.03\ddot{\theta}$ | M1A1 | | Equation of motion for pulley $rT = I\ddot{\theta}$<br>A1 for correct substitutions in each case |
| | $0.2T = \frac{0.03(9.8 - T)}{0.2}$ | | | |
| | $T = 4.2\,\text{N}$ | A1 | 5 | CAO |
| | **Alternative:** | | | |
| | $a = r\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{a}{0.2}$ | (M1) | | Connecting $a$, $\ddot{\theta}$ |
| | For bucket $u = 0,\ v = 2,\ s = \frac{5}{14}$ | | | |
| | using $v^2 = u^2 + 2as$ gives $a = \frac{28}{5} = 5.6$ | | | |
| | $\Rightarrow \ddot{\theta} = \frac{5.6}{0.2} = 28\ \text{rad s}^{-1}$ | (A1) | | |
| | Either $g - T = 0.2\ddot{\theta}$ or $0.2T = 0.03\ddot{\theta}$ | (M1A1) | | One other equation |
| | $T = 4.2\,\text{N}$ | (A1) | (5) | CAO |
| | **Total** | | **13** | |

**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **5(a)** |  | | | |
| | Moments about $O$: | | | |
| | $-3(2)-a(6)+2a(3)-4(5)+8(2)$ | M1<br>A2,1 | | One correct $\mathrm{F}\times d$ pairing seen<br>–1 each type of error |
| | $=-6-6a+6a-20+16$ | | | |
| | $=-10 \Rightarrow$ no '$a$' $\Rightarrow$ independent | A1 | | '$a$' cancels / comment about no '$a$' |
| | Magnitude = 10 | A1F | 5 | Must be positive value stated;<br>ft single slip |
| | **Alternative:** | | | |
| | $\mathbf{r}_1\times\mathbf{F}_1=\begin{vmatrix}\mathbf{i} & -1 & -8\\ \mathbf{j} & 2 & 0\\ \mathbf{k} & 0 & 0\end{vmatrix}=\begin{pmatrix}0\\0\\16\end{pmatrix}$ | (M1) | | One correct $\mathbf{r}\times\mathbf{F}$ seen |
| | $\mathbf{r}_2\times\mathbf{F}_2=\begin{vmatrix}\mathbf{i} & 2 & a\\ \mathbf{j} & 6 & -3\\ \mathbf{k} & 0 & 0\end{vmatrix}=\begin{pmatrix}0\\0\\-6-6a\end{pmatrix}$ | (A1) | | Second $\mathbf{r}\times\mathbf{F}$ correct |
| | $\mathbf{r}_3\times\mathbf{F}_3=\begin{vmatrix}\mathbf{i} & 3 & -4\\ \mathbf{j} & -5 & 2a\\ \mathbf{k} & 0 & 0\end{vmatrix}=\begin{pmatrix}0\\0\\6a-20\end{pmatrix}$ | | | |
| | Total moment $=16-6-6a+6a-20$ | (A1) | | $\sum$ ($k$ components) |
| | $=-10$ '$a$' cancels / no '$a$' | (A1) | | Comment about '$a$' |
| | Magnitude = 10 | (A1F) | (5) | ft single slip |
| **(b)(i)** | $\mathbf{F}=\begin{pmatrix}4\\-3\end{pmatrix}+\begin{pmatrix}-4\\8\end{pmatrix}+\begin{pmatrix}-8\\0\end{pmatrix}=\begin{pmatrix}-8\\5\end{pmatrix}$ | M1<br>A1 | 2 | Substitute $a=4$ and add forces |
| **(ii)** |  Moments about $O$: $-10=5d$ | M1 | | $y$-component $\times d$ = answer (a) |
| | $d=-2$ | A1F | 2 | ft (a) |
| | **Total** | | **9** | |

**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 6(a) |   Let density = $\rho$ <br> $m=\frac{4}{3}\pi r^3\rho \Rightarrow \rho=\frac{3m}{4\pi r^3}$ | B1 | | $\rho$, $m$ linked anywhere |
| | Mass of elemental disc / cylinder <br> $=\pi\left(r^2-x^2\right)\delta x\rho$ | M1 | | Attempt to use $\pi r^2 h\rho$ |
| | MI of elemental disc = <br> $\frac{1}{2}\left[\pi\left(r^2-x^2\right)\delta x\rho\right]\left[r^2-x^2\right]$ | A1 | | Use of $\frac{1}{2}mr^2$ |
| | MI of sphere = $\int_{-r}^{r}\frac{1}{2}\pi\rho\left(r^2-x^2\right)^2\,\mathrm{d}x$ | | | |
| | $=\frac{3m}{8r^3}\int_{-r}^{r}\left(r^4+x^4-2r^2x^2\right)\mathrm{d}x$ | m1 | | Attempt at integral, dep on first M1 |
| | $=\frac{3m}{8r^3}\left[r^4x+\frac{x^5}{5}-2r^2\frac{x^3}{3}\right]_{-r}^{r}$ | A1F | | Their expression integrated correctly – correct number of terms only |
| | $=\frac{3m}{8r^3}\left[2\left(r^5+\frac{r^5}{5}-\frac{2r^5}{3}\right)\right]$ | | | Correct limit used, $\rho$ replaced to obtain answer given |
| | $=\frac{3m}{4r^3}\times\frac{8r^5}{15}=\frac{2}{5}mr^2$ | A1 | 6 | AG |
| | **Alternative:** | | | |
| |   Let density $=\rho$ <br> $m=4\pi r^3\rho$ <br> $\Rightarrow\rho=\frac{3m}{4\pi r^3}$ | (B1) | | $\rho$, $m$ linked anywhere |
| | Mass of elemental shell $=4\pi x^2\delta x\rho$ | (M1) | | Attempt to use surface area of sphere $\times\,\delta x\times\rho$ |
| | MI of elemental shell about diameter <br> $=\frac{2}{3}\left(4\pi x^2\delta x\rho\right)x^2$ | (A1) | | Use of $\frac{2}{3}mr^2$ |
| | MI of sphere $=\int_{0}^{r}\frac{8}{3}\pi x^4\rho\,\mathrm{d}x$ | | | |
| | $=\frac{2m}{r^3}\int_{0}^{r}x^4\,\mathrm{d}x$ | (m1) | | Attempt at integral; dep on first M1 |
| | $=\frac{2m}{r^3}\left[\frac{x^5}{5}\right]_{0}^{r}$ | (A1F) | | Their expression integrated correctly |
| | $=\frac{2mr^5}{5r^3}$ | | | Correct limits used, $\rho$ replaced |
| | $=\frac{2mr^2}{5}$ | (A1) | (6) | AG |

**MM04 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **6(b)(i)** | $\text{MI}_{\text{ROD}} = \frac{4}{3}(3m)(2l)^2 = 16ml^2$ | B1 | 1 | Use of $\frac{4}{3}ml^2$ with '$m$' = $3m$ and '$l$' = $2l$ |
| **(ii)** | $\text{MI}_{\text{SPHERE (about } G)} = \frac{2}{5}(5m)(l)^2 = 2ml^2$ | B1 | | Use of $\frac{2}{5}ml^2$ with '$m$' = $5m$ and '$l$' = $l$ |
| | $\text{MI}_{\text{SPHERE (about } P)} = 2ml^2 + 5m(5l)^2$ | M1 | | Use of parallel axis theorem |
| | $= 127ml^2$ | A1 | | |
| | $\text{I}_{\text{PENDULUM}} = 127ml^2 + 16ml^2 = 143ml^2$ | A1 | 4 | AG |
| **(iii)** | Angular momentum of clay before collision = $mv(3l) = 3mvl$ | B1 | | |
| | Angular momentum for pendulum after collision $= \text{I}_1\omega = 143ml^2\omega$ | B1 | | Correct for pendulum |
| | Angular momentum for clay after collision = $\text{I}_2\omega = m(3l)^2\omega$ | B1 | | Correct for clay |
| | Conservation of angular momentum: $3mvl = 143ml^2\omega + 9ml^2\omega$ | M1A1 | | Attempt at conservation of momentum |
| | $3mvl = 152ml^2\omega$ | | | |
| | $\omega = \frac{3v}{152l}$ | A1F | 6 | ft one slip |
| | **Total** | | **17** | |
| | **TOTAL** | | **75** | |